Unit 15.5
Day 2
Section 4.5
Rational Functions
Part 1 - Finding Asymptotes

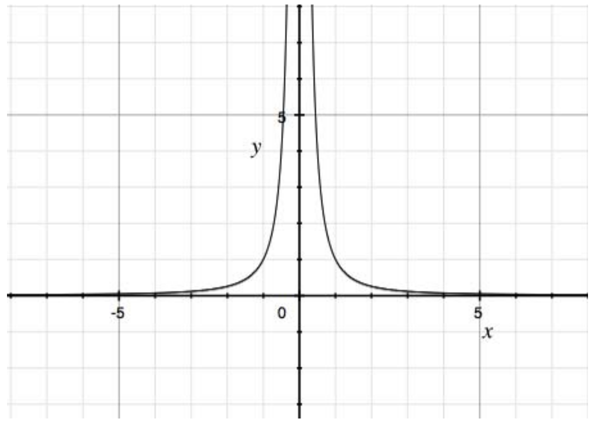
RATIONAL FUNCTIONS

If $p(x)$ and $q(x)$ are polynomials with $q(x) \neq 0$,
then

$$f(x) = \frac{p(x)}{q(x)}$$

defines a RATIONAL FUNCTION.

EXAMPLES: $h(x) = \frac{x-2}{x}$ $\qquad$ $f(x) = \frac{2x}{x^3 - 8}$



In the graph above, the line x = 0 is a vertical asymptote and the line y = 0 is a horizontal asymptote.

Types of asymptotes

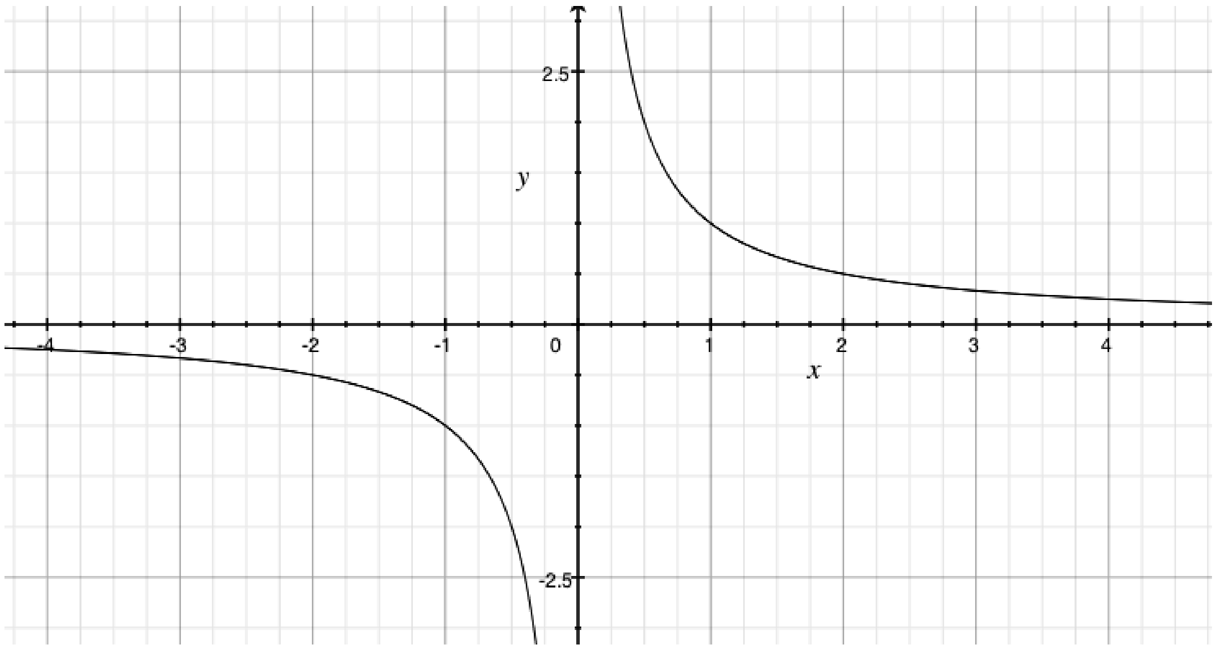
1) Vertical asymptotes: is/are vertical line(s) that the graph will NEVER cross. They have an "x = constant" equation.

The x values that make the function undefined.

2) Horizontal asymptotes: is a horizontal line that the value of the function approaches as $x \to \infty$ or as $x \to -\infty$. It has a "y = constant" equation. The graph of a function MAY cross a horizontal asymptote.

3) Oblique asymptotes: are neither horizontal or vertical. They have a "y = mx + b" equation. The graph of a function MAY cross a oblique asymptote.

What are the vertical and horizontal asymptotes of this graph and why do they occur?



Finding vertical asymptotes: Set the denominator equal to zero to find the values of x that make the function undefined. Remember to reduce the fraction first (if possible).

1) $g(x)=\frac{x+1}{2x^2+5x-3}=\frac{x+1}{(2x-1)(x+3)}$ $x=\frac{1}{2},\ x=-3$

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2) $f(x)=\frac{x^2-1}{x^2+3x+2}=\frac{\cancel{(x+1)}(x-1)}{(x+2)\cancel{(x+1)}}$ $x=-2$

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3) $h(x)=\frac{3}{-x-5x^2-4x}=\frac{3}{-5x^2-5x}=\frac{3}{-5x(x+1)}$

$x=-1,\ x=0$

FINDING HORIZONTAL ASYMPTOTES

1) If the degree of the numerator = degree of the denominator,
then the horiz. asymptote equals the ratio of the leading terms.

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2) If the degree of the num. < degree of the denominator,
then the horiz. asymptote y = 0.

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3) If the degree of the numerator - degree of the denominator = 1
then divide the num. by den. The quotient is called the oblique asymptote
(ignore remaider).

$y = mx + b$

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4) If the degree of the num. - degree of the denominator > 1
then there is no horizontal asymptote.

Finding horizontal asymptotes:

1) $f(x)=\dfrac{x+1}{2x^2+5x-3}$ $y=0$

2) $f(x)=\dfrac{2x^2-3x+1}{5x^2+6x+1}$ $y=\dfrac{2}{5}$

3) $f(x)=\dfrac{x^2+7x+6}{x+4}$

$$
\begin{array}{r}
x+3 \\
x+4 \overline{\smash{)}\, x^2+7x+6} \\
-(x^2+4x) \\
\hline
3x+6 \\
-(3x+12) \\
\hline
-6
\end{array}
$$

$y=x+3$

4) $g(x)=\dfrac{-3x^6-7}{x^3}$ No horiz. asymptote

Homework: p. 326: 1-8, 15-22, plus asymptote worksheet